Appendix A: Data acquisition

The Runge-Kutta method is a numerical technique used for solving ordinary differential equations (ODEs), such as those governing the behavior of the Duffing oscillator. It's commonly employed when analytical solutions are difficult to obtain. Here's an explanation of how the Runge-Kutta method can be applied to solve the Duffing oscillator equation in third person, including mathematical notation:

The Duffing oscillator is described by the second-order ordinary differential equation:

Where:

* is the stiffness constant.
* is a coefficient that determines the strength of nonlinearity.
* is the displacement of the oscillator at time .
* is the damping coefficient.
* represents the first derivative of with respect to time, which is velocity.
* is the mass of the oscillator.
* represents the second derivative of with respect to time , which is acceleration.
* is the external force applied to the oscillator at time .

To apply the Runge-Kutta method to solve this equation, by convert this second-order ODE into a system of first-order ODEs. and introducing a new variable, such as , to represent the velocity . Then, two first-order ODEs become:

To solve this equation using the Runge-Kutta method, the following steps are typically followed:

1. Discretization: Divide the time interval over which you want to solve the equation into small time steps. Let represent the size of each time step, and create a time grid with time points where .
2. Initialization: Set the initial conditions for the displacement and velocity, and ​, at ​.
3. Iteration: For each time step , perform the following calculations:
   1. Calculate the acceleration at time ​ using the Duffing oscillator equation [reference to Duffing equation]
   2. Use the Runge-Kutta method to update the displacement and velocity for the next time step:
   3. Update the displacement and velocity for the next time step:
4. Repeat step 3 for each time step until you reach the desired endpoint.

The Runge-Kutta method iteratively approximates the solution to the Duffing oscillator equation by considering the rate of change of displacement and velocity at each time step, providing a numerical solution for and over the specified time interval.